Robust Discretization of Flow in Fractured Porous Media

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Key words: mixed finite element, mortar finite element, nonconforming grids, fractured media.

ABSTRACT

Fractures are ubiquitous in natural rocks, and in many cases have a leading order impact on the structure of fluid flow [5]. As such, detailed and robust modeling of coupled flow between fractures and a permeable rock is essential in applications spanning from enhanced geothermal systems, via CO_2 storage to petroleum extraction. Since fractures frequently have aspect ratios as high as 100-1000, it is appealing to consider them as lower-dimensional features [1]. In this context, we then consider three-dimensional domain of permeable rock, within which (multiple) fractures will be represented by (multiple) two-dimensional manifolds. In the case where two or more fractures intersect, we will naturally also be interested in the intersection lines and points.

Due to the complex structure of natural fracture networks [1], it remains a challenge to provide robust and flexible discretization methods. Here, we identify a few distinct features which are attractive from the perspective of applications.

First, we emphasize the importance of mass conserving discretizations. This is of particular significance when the flow field is coupled to transport (of heat, or composition), as transport schemes are typically very sensitive to non-conservative flow fields. The second property of interest is grid flexibility. This is important both in order to accommodate the structure of the fracture network, but also in order to honour other properties of the problem, such as material heterogeneities or anthropogenic features such as wells. Finally, we prefer methods which are robust in the physically relevant limits. In the case of fractures, it is imperative to allow for arbitrarily large aspect ratios, that is to say, thin fractures.

Several methods have been proposed to discretize fractured porous media, including methods based on the mortar method [4] and X-FEM approaches [2]. However, to our knowledge, no method has been presented which fullfills the above properties, and which is amenable to rigorous analysis.

We propose a new method, based on the structure of mortar methods [3]. Our formulation is novel in that it uses a mixed formulation in the mortar space, consisting of tangential fluxes and pressure, which are coupled to the surrounding domain via jumps in traces of the external fluxes. We formulate the method hierarchically, which allows for a unified treatment of the permeable domain, the fractures, intersection lines and intersection points. The mixed formulation ensures that the discretization is conservative, and we also explicitly derive a finite volume variant of our





Figure 1: Results for the pressure (top) and velocity (bottom) which show that the method is capable of handling immersed and crossing fractures as well as fracture intersections. The behaviour of the fractures with respect to the flow is changed when the relevant parameters are altered.

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