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Stabilized X-FEM for Heaviside and Nonlinear Enrichments

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ABSTRACT

The extended Finite Element Method, introduced by Ted Belytschko and coworkers and reviewed in [1], is considered nowadays an almost mature technology, so that various commercial implementations exist. Nonetheless, there are several fundamental aspects normally overlooked, that can be considerably improved in view of large scale applications. Typical examples are blending to standard finite elements, where effective solutions have been found, and efficient quadrature in enriched elements that is still an open problem although many contributions exist.

Among these fundamental aspects, a major one is played by the so-called ”stabilization”, i.e. a remedy to the fact that, in some circumstances, the X-FEM formulation turns out into an indeterminate system of equations. This is evidenced by ill-conditioning during linear equations solving.

For the Heaviside function enrichment, representing displacement jumps, this problem arises when the discontinuity gets close to enriched nodes. However, this is a general problem also when nonlinear enrichments, like crack tip fields or regularized jumps, are used in conjunction with the concept of geometric enrichment [2]. In fact, toward the edges of the enriched area, where the nonlinearity attenuates, its numerical similarity to a linear function causes a blow up in the condition number.

Although this problem arises frequently in practical applications, it is surprisingly faced by few publications. Proposed techniques for its solution include modification of the enrichment structure [3], use of system preconditioners, eigenvalue decomposition of the element stiffness matrix [4]. The proposed literature approaches are based on the idea of correcting the enrichment space to solve the problem. However, this is sometimes of difficult implementation and sometimes complexifies the approximation space, with the consequence of requiring an higher computational effort.

An analytic study of the problem has been done to see how the structure of the problem changes in the above degenerate cases. Then, to ensure optimal conditioning, computational speed in large scale applications and to keep the lightweight structure of the original X-FEM enrichment, the enrichment variables are constrained by small penalty terms in the global variational problem. This has the consequence that:

- the approximation space is left unmodified;
- the stabilization can be applied after the global system matrix assembly, with no impact on the computational performance of the method;
- the stabilization applies unmodified for any space dimensions and finite element family.

This contribution presents the development of this new stabilization approach as well as the results on some benchmark problems, ranging from 1D to 3D. It is shown how the proposed stabilization markedly improves the condition number of the system of equations, both in the case of Heaviside and nonlinear enrichments. To give an esemplificative example, Figure 1, shows the displacement contours of a 3D bar with prescribed ends displacement subjected to a slant cut and discretized with a mesh of 14440 hexahedral elements. This problem cannot be solved without stabilization as the linear system solver exits with zero pivot error detection and (numerically) infinite condition number. With the proposed method, the problem condition number shows a very limited variation: from 5.9E+05 in the linear elastic case (no cut discontinuity) to 2.0E+06 when the cut is introduced. This depicts the effectiveness of the new stabilization method.

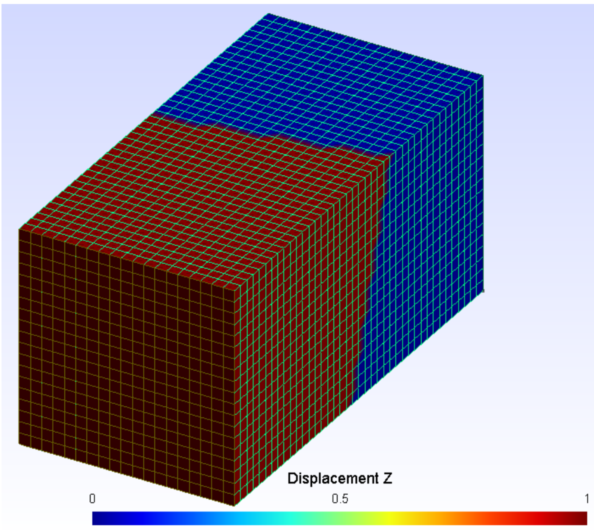

Figure 1: A 3D parallelepiped with a slant cutting plane and prescribed displacements at the ends.

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