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Can h-multigrid redeem coupled variables dG discretizations of the incompressible Navier-Stokes equations?

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ABSTRACT

Discontinuous Galerkin (dG) methods have proved to be effective in the CFD field allowing to simulate complex physics in complex domains without compromise in terms of accuracy and stability. High order accurate discretizations can be easily devised increasing the polynomial degree and robustness in convection dominated flows is guaranteed by good local conservation properties. Nowadays dG methods are mainly applied for the simulation of compressible fluid flows, being the gold standard in many challenging aerospace and aeroacoustic applications [1]. As opposite their adoption by incompressible fluid flow practitioners is still limited due to challenges involved in the numerical solution of the Incompressible Navier-Stokes (INS) equations. On the one hand explicit time integration strategies result in poor pressure accuracy reducing the appeal of high order accurate spatial discretizations. On the other hand fully implicit fully coupled velocity-pressure spatial discretisations result in systems of Differential Algebraic Equations (DAEs) that are very expensive to solve due to the indefiniteness of the resulting system matrices, their poor spectral properties, and the saddle point nature of the problem [2].

During the last couple of decades efficiency was often pursued by means of decoupled time integration strategies, see *e.g.* [3]. These formulations allow to solve the momentum equation in a segregated manner (forgetting about the divergence free constraint this amounts at solving a standard convection-diffusion problem) and to exploit some discrete form of the Pressure Poisson Equation (PPE) to recover incompressibility. Decoupled equations are easier to solve employing standard iterative solvers but there are several pitfalls: (i) pressure is still a global unknown, (ii) pressure accuracy is limited to second order in time, (iii) incompressibility is only weakly enforced, (iv) the boundary condition treatment is not trivial.

In an attempt to overcome these limitations we propose to exploit h-multigrid strategies to efficiently solve coupled variables dG discretizations of the INS equations. To accelerate the convergence of basic iterative methods multigrid strategies rely on global corrections accomplished by solving a series of coarse problems. In the context of dG formulations coarse discretizations are explicitly built over a hierarchical sequence of nested grids that might be generated recursively by agglomeration of a fine grid [4, 5]. While this approach is flexible enough to account for complex 3D domains, polyhedral elements of very general shape must be dealt with, a prerogative of agglomeration based dG discretizations [6].

Several theoretical and practical aspects of geometric multigrid are investigated considering the application to linear and non-linear model problems.

- Following [7] we analyze in detail the performance of the BR2 dG discretization [6] with inherited or non-inherited coarse grid operators. The latter are obtained assembling the weak form terms on every sub-level while the former are constructed restricting the fine grid operator.
- The computational cost of building coarse grid operators of inherited and non-inherited type is investigated. The latter require to numerically integrate weak forms on agglomerated elements at each iteration of a non-linear solver strategy while the former relies on precomputed inter-grid transfer operators.
- We perform numerical test cases evaluating the influence of the number of sub-levels and of the polynomial degree on the convergence rate.
- The performance of multigrid as a standalone solution strategy or as a preconditioner for Krylov solvers is compared.
- The scalability issue in large scale parallel computations is also explored.

The best performing approaches are compared with state of the art iterative solvers considering both elliptic problems and incompressible fluid flow problems.

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