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Three-Dimensional Crack Propagation with Global Enrichment XFEM and Vector Level Sets

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ABSTRACT

Three-dimensional (3D) crack propagation problems are solved by combining a three dimensional version of the vector level set method [1] with a variant of the extended finite element method, which enables the use of enriched elements in a fixed volume around the crack front. The combination of the two methods yields several advantages both in terms of accuracy and in terms of computational cost and ease of implementation.

One major problem associated with the use of geometrical enrichment in three-dimensional fracture problems is the increase in the condition numbers of the resulting system matrices, which results in very large solution times and, in some cases, even in systems that are practically unsolvable. The proposed method, which can be regarded as a 3D extension of the degree of freedom gathering technique [2], overcomes this problem by discretizing the crack front with special univariate elements whose shape functions serve as a basis with which the tip enrichment functions are weighted. The use of this special basis inhibits spatial variation of the tip enrichment functions in the plane normal to the crack front, which is the cause of the conditioning problems, while still allowing variation along the crack front which is essential to reproduce the behavior of solution variables, such as stress intensity factors, in general 3D problems. By employing this global tip enrichment technique it is possible to use tip enriched elements in a fixed volume around the crack front, which leads to optimal convergence rates, while keeping the conditioning of the resulting systems of equations almost unaffected.

An additional advantage of the proposed enrichment technique is that it can be combined, in a straightforward way, to techniques for minimizing blending errors between the enriched and the standard part of the approximation [3, 4] resulting in a further increase in the accuracy of the resulting method.

As far as crack representation is concerned, the three-dimensional vector level set method [1] is employed which constructs level set data for propagating crack surfaces by employing only geometric and vector operations, thus avoiding the solution of differential evolution equations.

More specifically, the crack front at any given time is defined as an ordered sequence of line segments which is in turn defined by a set of points that lie on the crack front. Once the advance vectors for those points are known, the new location of the crack front can be obtained. In addition, the crack surface advance can be represented geometrically by a sequence of four sided bilinear surfaces whose vertexes are the points lying on the initial and updated crack front. These surface elements are used to compute the level set functions.

The combination of the aforementioned methods is quite convenient since both methods employ a set of points and line segments/elements in order to discretize the crack front. As a result, the same crack front discretization can be used for both methods. The resulting methodology can handle general crack propagation problems providing high accuracy, reduced computational cost and simple implementation. Finally, as a direction of future work, the method could be coupled to one of the available error estimators [5, 6] and applied to more complex industrial problems [7].

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