X-DMS 2015
eXtended Discretization MethodS
September 9-11, 2015, Ferrara, Italy

Numerical approximation of coupled PDEs with high dimensionality gap

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Key words: Dimensional model reduction, networks and heterogeneous media, tumor microenvironment, fractured reservoirs.

ABSTRACT

The simulation of multiscale, multiphysics, multimodel systems is among the grand challenges in Computational Science & Engineering. In this context, the application of dimensional (or topological) model reduction techniques plays an essential role. For example, small inclusions of a continuum can be described as zero-dimensional (0D) or one-dimensional (1D) concentrated sources in order to reduce the computational cost of simulations. However, concentrated sources lead to singular solutions that still require computationally expensive graded meshes to guarantee accurate approximation. Many problems in this area are not well investigated yet, such as the coupling of three-dimensional (3D) continua with embedded (1D) networks, although they arise in applications of paramount importance such as microcirculation, flow through perforated media and the study of reinforced materials. We will shed light on these unexplored mathematical problems by casting them in a new unified framework to formulate and approximate coupled partial differential equations (PDEs) on manifolds with heterogeneous dimensionality arising from topological model reduction. The main computational barrier consists in the ill-posedness of restriction operators (such as the trace operator) applied on manifolds with co-dimension larger than one.

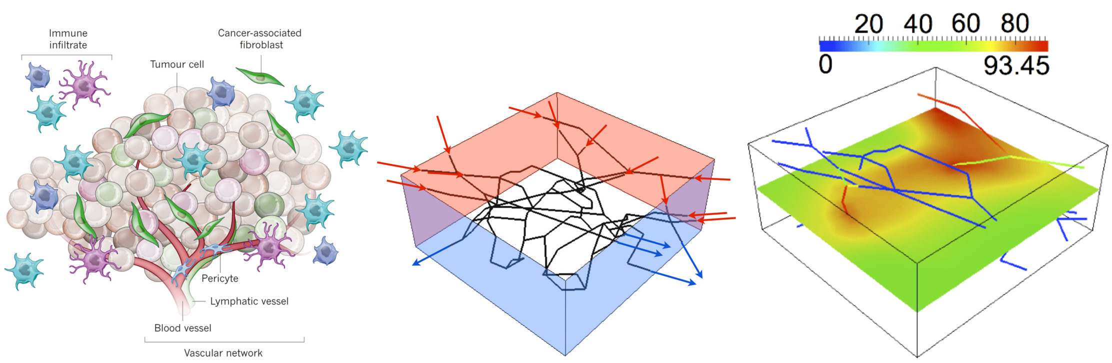

Figure 1: A sketch of the tumor microenvironemnt (left). Geometrical configuration of microvasculature (middle). Simulation of anticancer drug profiles $[g/cm^3]$ (right) after injection of drug loaded nanoparticles into the vascular network [1, 2, 3].

Our ultimate objective is to exploit topological model reduction to perform large scale simulations of biomechanics of tumor micro-environment and geomechanics of fractured reservoirs

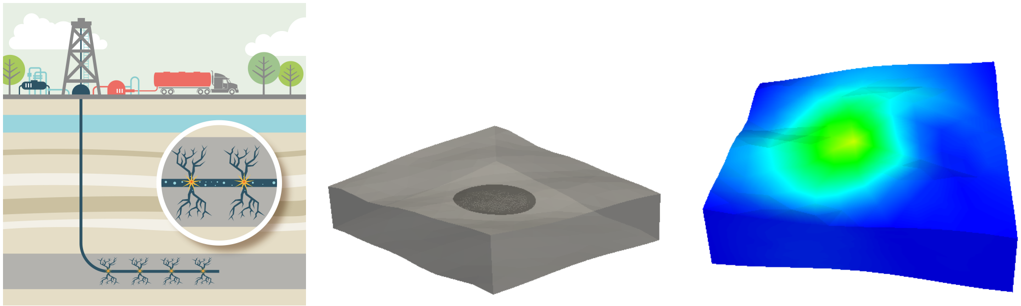

Figure 2: A sketch of hydraulic fracturing. Simulation of surface uplift after injection of fluid into a circular fracture [4].

(see Figure 1 and 2 for a sketch of the problems and related simulations). We will apply the proposed method to address relevant questions for cancer treatment by means of different approaches, such as chemotherapy of hyperthermia. Similar considerations apply to geomechanics, where we will enable to combine state of art fracture propagation models with large scale reservoir simulation, for a better exploitation of the geological resources and safer control of the environmental impact.

We will overcome the computational challenges of approximating PDEs on manifolds with high dimensionality gap by means of nonlocal restriction operators that combine standard traces with mean values of the solution on low dimensional manifolds. This new approach has the fundamental advantage to enable the approximation of the problem using Galerkin projections on Hilbert spaces, which can not be otherwise applied because of regularity issues. The corresponding error analysis will naturally inform about the concurrent modeling and discretization errors in the approximation of the original fully dimensional problem. We call this new computational approach the geometric embedded multiscale method (GEMME).

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