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The fat boundary method for the Stokes problem

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ABSTRACT

The **Fat Boundary Method** (**FBM**), introduced in [2], is a fictitious domain method for solving partial differential equations in a domain with holes. The typical situation, which is met for instance in the context of fluid particle flows, is that of a perforated domain, $\Omega = \Theta \setminus \overline{B}$, where Θ is a simple shaped domain, say a cube, and B is a collection of (possibly many) smooth open subsets (the holes). The method consists in splitting the initial equations into two problems to be coupled via Schwartz type iterations: the solution of a global problem set in Θ, for which we assume that fast solvers can be used, and the solution, fully in parallel, of a collection of independent local problems defined on an auxiliary domain ω composed by narrow strips around the connected components of B (the so called *fat boundary*). The coupling between the global problem and the local ones is based on the one hand on the interpolation of a globally defined field on the artificial boundary γ' which together with ∂B delimits the auxiliary domain ω, and on the other hand on the prescription of a jump in the normal derivative across the boundary of B. For example, the Poisson equation $-\Delta u = f$ in Ω, with homogeneous Dirichlet boundary conditions on $\partial\Omega$ is reformulated as

$$\begin{cases} a: \quad -\Delta v = f \quad \text{in } \omega, \qquad v = 0 \text{ on } \partial B, \quad v = \widehat{u} \text{ on } \gamma' \\ \\ b: \quad -\Delta \widehat{u} = \overline{f} + \dfrac{\partial v}{\partial n}\delta_\gamma \quad \text{in} \quad \Omega, \end{cases} \tag{1}$$

where $\bar{f}$ is the extension of f to Θ with $\bar{f} = 0$ in B. It is in fact not difficult to prove that the solution u of the Poisson equation coincides with the restriction to Ω of $\widehat{u}$. Moreover, while most fictitious domain methods result in a degradation in the accuracy in comparison with boundary fitted methods, under suitable assumptions the FBM retains optimality, even when using high order discretizations. More precisely it is possible to show that *if u is sufficiently smooth in $\Omega = \Theta \setminus \bar{B}$, domain of definition of the original problem*, then the FBM achieves the best order of approximation allowed by the chosen approximation spaces ([1]).

We will extend the method to the Stokes equation starting from an equivalent formulation of (1) in the form of a saddle point problem, where the coupling between a) and b) is expressed

in a weak form via Lagrange multipliers. For the resulting FBM formulation of the Stokes problem in the continuous framework, we will prove well posedness and equivalence with the standard formulation. We will present two possible Galerkin discretizations: for the first one particular attention has to be paid to the mesh sizes used for the different unknowns (local and global velocity and pressure). For the second formulation a stabilization term will be added to the local problem, which will give us more freedom in the choice of the different mesh-sizes. Stability and convergence analysis will be presented for both formulations.

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