X-DMS 2015
eXtended Discretization MethodS
September 9-11, 2015, Ferrara, Italy

XFEM vs. r-Refinement for Higher-Order Approximations in Elastoplasticity

Samir Omerović[1] and Thomas-Peter Fries[2]
[1] Graz University of Technology, Lessingstraße 25/II, samir.omerovic@tugraz.at
[2] Graz University of Technology, Lessingstraße 25/II, fries@tugraz.at

Key words: XFEM, r-Adaptivity, DG-XFEM, Elastoplasticity, Weak discontinuities, Interface capturing

ABSTRACT

From analytical and semi-analytical solutions for elastoplastic problems, see [1], it is known that the displacement field as well as the stress field, may feature weak discontinuities. These discontinuities occur in the transition zone between elastic and elastoplastic behaviour, as shown in [2]. Under certain conditions, e.g. perfectly plastic material and specific loading, even strongly discontinuous behaviour appears, see [3]. It is important to emphasize that the location of the elastic-plastic interface is not known beforehand but it is part of the solution. The fact that the elastoplastic interface is within the elements drastically reduces the convergence rates, as shown in the figure below. This has also implications for the application of adaptive schemes.

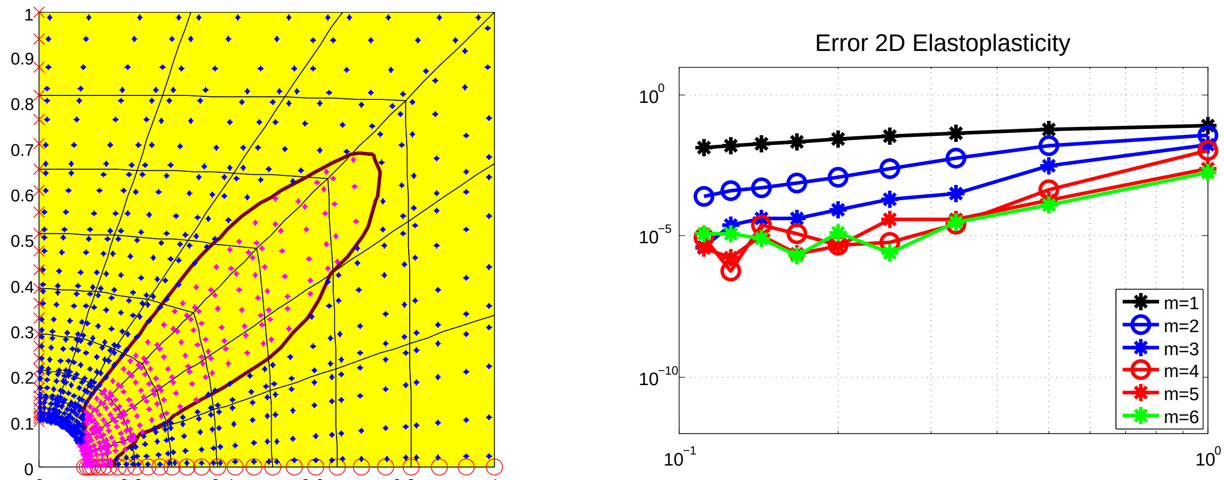

Figure 1: Error in deformation energy for an elastoplastic system

The common approach to adaptively improve convergence properties, is to change element size (h-refinement), to increase the polynomial degree of the ansatz functions (p-refinement), or to reallocate the nodes (r-refinement), see also [4]. Each of these methods differs in terms of computational costs and accuracy. Our emphasis is on convergence rates and in this case the p-refinement seems to be the optimal choice. However, this is true for smooth displacement fields only. As aforementioned, for elastoplasticity, a kink in the displacement field occurs, and for such cases additional considerations have to be made.

We propose two different approaches for this problem, one procedure uses the extended finite element method (XFEM), the second approach is a tailored version of r-refinement for weak discontinuities.

The first method relies on the properties of the extended finite element method to include known solution properties into the approximation space. Our suggestion is to enrich the approximation space in the presence of weak discontinuities at the plastic interface with modified abs functions, see [5]. As commonly executed within the framework of the XFEM, the plastic interface is tracked using the level set method, e.g. in [6]. It is stressed that no remeshing whatsoever is necessary. Another approach, also within the XFEM-framework, is to use Heaviside enrichment and enforce continuity using Lagrange multipliers. The numerical examples confirm that with minimal effort, the absolute error is significantly reduced.

The second method changes the position of the nodes in a tailored iterative procedure and also adds additional elements if required. Within this scheme, the zero of the level set function is used to represent the elastoplastic interface. If an element is crossed by the zero level set, an Newton-Raphson scheme is utilized to either subdivide the element and align the element edges along the interface or to move the nodes in a way that the interface is represented in an optimal way. The emphasis in this approach is on the reconstruction of the higher order level-set and on the integration of the weak form. The blending function method, see [7], is used to map elements with a higher order side such that the higher order face align with the elastoplastic interface.

REFERENCES

[1] Lubliner J. *Plasticity Theory*. Dover Publications, (2008).

[2] Nübel V., Düster A., Rank E. An rp-adaptive finite element method for the deformation theory of plasticity. *Computational Mechanics*. 557–574, 39(5) (2007).

[3] Johnson C. and Scott R. A Finite Element Method for Problems in Perfect Plasticity Using Discontinous Trial Functions. *Nonlinear Finite Element Analysis in Structural Mechanics*. 307–324, (1981).

[4] Wriggers P. *Nonlinear Finite Element Methods*. Springer, (2008).

[5] Moës N., Cloirec M., Cartraud P., Remacle J.-F. A computational approach to handle complex microstructure geometries. *Comput. Method Appl. M*. 3163–3177, 192 (2003).

[6] Fries T.-P., Belytschko T. The extended/generalized finite element method: An overview of the method and its applications. *Int. J. Numer. Meth. Engng*. 253–304, 84(3) (2010).

[7] Szabó B.A. and Babuška, I., *Finite Element Analysis*. Wiley, (1991).