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# A Dynamic Conformal Decomposition Finite Element Method with Guaranteed Quality

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## ABSTRACT

Enriched finite element methods such as the Generalized Finite Element Method (GFEM), the eXtended Finite Element Method (XFEM), and the Conformal Decomposition Finite Element Methods (CDFEM) are powerful tools for multiphase and multimaterial problems. These methods provide discretizations that dynamically adapt to the moving material and phases to accurately capture the interfacial physics and discontinuities. In order to allow for a discontinuous description across material or phase interfaces, these methods introduce some level of enrichment in the elements that are crossed by these interfaces. Additional unknowns are assigned to one or more of the mesh entities (elements, nodes, sides, or edges) that are associated with these interfacial elements, and additional equations are formulated for these unknowns.

Care must be taken in all enriched finite element methods to make sure that the resulting system of equations is well conditioned. As an interface can come arbitrarily close to background mesh nodes, the equations for the added degrees of freedom may become linearly dependent on the equations for the existing degrees of freedom. To handle this issue, practitioners have omitted the enrichment in elements that intersect only a small fraction of the additional material [1]. In this way the interface can be considered to be snapped to the nodes of the background mesh when it comes very close to these nodes.

The Conformal Decomposition Finite Element Method (CDFEM) is an enriched finite element method that can be used to describe arbitrarily discontinuous physics across dynamic interfaces. A level set is used to describe the location of the moving interface. Nodes are added at the intersection of the level set surface with the edges of the input mesh, and a conforming mesh is generated automatically. Standard unstructured mesh data structures are generated for the resulting conformal mesh in terms of element blocks and side sets. This general framework allows the physics code to describe either weak or strong discontinuities across the interface using standard finite element methods.

Like other enriched finite element methods, CDFEM can produce poorly conditioned systems of equations when the interface comes arbitrarily close to the background mesh nodes. This poor conditioning can be addressed by snapping the interface to the nearest node when the edges of the background mesh are cut by the interface near the ends of the edge. However, this snapping introduces an error in the location of the interface. In the current work, an alternate approach is taken that removes the poor conditioning without introducing an error in

the interface location. When an edge is crossed near one of its ends, the nearest node of the edge is moved to the crossing, instead of moving the crossing to the node, as shown in Figure 1. This method is similar to that used in Isosurface Stuffing [2]. This approach improves the quality of the resulting decomposed meshes and produces discrete systems of equations with dramatically better conditioning. Because no error is introduced in this process, larger snapping tolerances can be used to produce high accuracy and robustness. Earlier work [2] has shown that the quality of the resulting discretization can be guaranteed. The resulting system of equations is robustly and efficiently solved using traditional linear solvers. The method is described further and the improvement in the matrix conditioning is quantified.

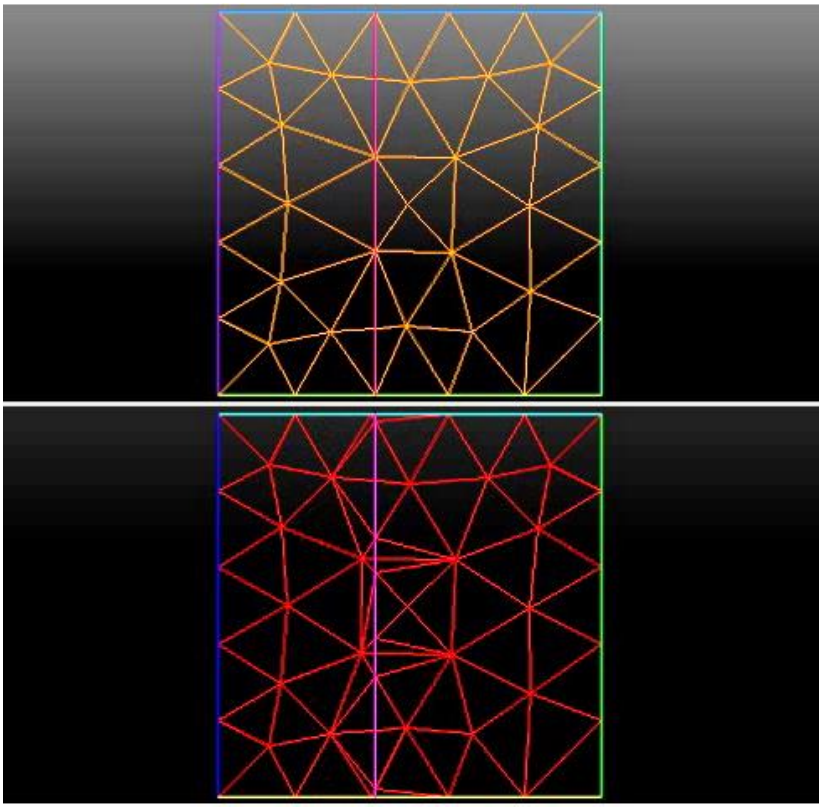

Figure 1. Comparison of meshes produced by Conformal Decomposition Finite Element Method (CDFEM) with edge snapping to produce guaranteed quality discretizations (top) versus traditional CDFEM enrichment (bottom).

## REFERENCES

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