

Simulation of damage and crack propagation at re-entrant corners according to the phase field method

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ABSTRACT

The detection of failure mechanisms in structures due to crack initiation and growth via numerical modeling is of great importance in engineering applications and has continually been the subject of attention by many researchers. The fundamental importance is the crack-tip field, i.e., stresses and strains close to a crack tip. The geometry of a plane elastostatic problem of a re-entrant corner shown in Fig (b) made of an isotropic, linearly elastic material under static loading will be investigated according to asymptotic methods [1] to characterize the singular stress field at the wedge tip. Propagating cracks can exhibit a rich dynamical behavior controlled by a subtle interplay between microscopic failure processes in the crack tip region and macroscopic elasticity.

The phase-field method has emerged as a powerful method to simulate crack propagation [2]. This method, developed originally for phase transformations, has the well-known advantage of avoiding explicit front tracking by making material interfaces spatially diffuse. In a fracture context, this method is able to capture both the short-scale physics of failure and macroscopic linear elasticity within a self-consistent set of equations that can be simulated on experimentally relevant length and time scales.

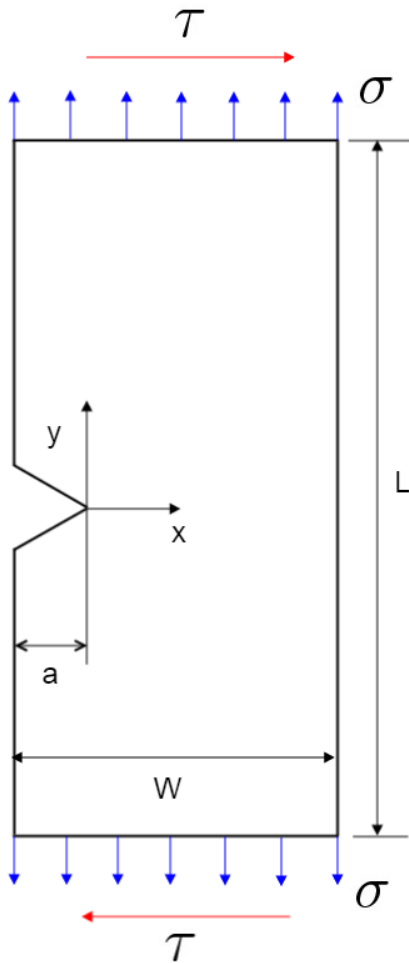
$$E(\mathbf{u}, \Gamma) = \int_{\Omega} \psi_o(\varepsilon(u)) \, d\mathbf{x} + G_c \int_{\Gamma} ds \quad (1)$$

The above equation governs process of crack initiation, propagation and branching by minimization problem of energy functional $E(\mathbf{u}, \Gamma)$, where ψ_o is elastic energy density function and G_c is Material fracture toughness.

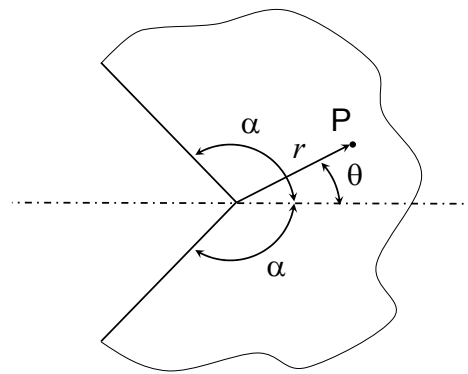
$$E_{\epsilon}(\mathbf{u}, s) = \int_{\Omega} (s^2 + \eta) \psi_o(\varepsilon(u)) \, d\mathbf{x} + G_c \int_{\Omega} \left(\frac{1}{4\epsilon} (1 - s)^2 + \epsilon |\nabla s|^2 \right) d\mathbf{x} \quad (2)$$

The phase field model is formulated as coupled dynamical equations $E_{\epsilon}(\mathbf{u}, s)$ for the phase (s) and displacement fields (\mathbf{u}) that constitute the weak form the finite element implementation in FEAP. The width of this transition zone (ϵ), which surrounds the phase field cracks, is controlled by a regularization parameter (η).

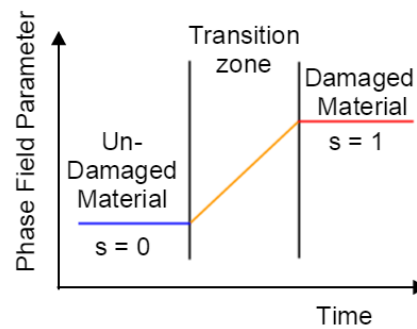
The fracture is indicated by a scalar order parameter (s), which is coupled to the material properties in order to model the change in stiffness between damaged and undamaged material [2]. When $s = 0$, the material is undamaged, while $s = 1$ for damaged material (cracked surface) refer Fig (c). At interfaces between damaged and undamaged material, the order parameter interpolates smoothly between the values assigned to the different material phases. This contribution will discuss and quantify with useful correlations the effect of the power of the stress singularity at the tip of a re-entrant corner –modified by varying the re-entrant corner angle– on the spatial distribution of the phase field variable s .



(a) Geometry of Mixed Mode problem



(b) Sketch of a sharp re-entrant corner



(c) Phase transition from un-damaged to damaged material

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