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Efficient Discontinuous Galerkin methods with local time stepping for immersed boundary problems

Stephan Krämer-Eis*[1,2], Björn Müller[1,2], Martin Oberlack[1,2,3]
[1]TU Darmstadt, Chair of fluid dynamics, Otto-Berndt-Str. 2, D-64287 Darmstadt
[2] TU Darmstadt, Graduate School Computational Engineering, Dolivostr. 15, D-64293 Darmstadt
[3] TU Darmstadt, Center of Smart Interfaces, Alarich-Weiss-Straße 10, D-64287 Darmstadt
kraemer-eis@fdy.tu-darmstadt.de, mueller@fdy.tu-darmstadt.de, oberlack@fdy.tu-darmstadt.de*

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ABSTRACT

In this talk, we study the flow of inviscid compressible fluids in domains with immersed interfaces. In particular, we propose an optimally convergent extension of the Discontinuous Galerkin (DG) method that incorporates a level set function whose zero iso-contour defines a sharp boundary or interface. Two main challenges arise in the development of such a numerical method: The integration of higher order polynomials over complex domains which are at least partly defined implicitly, and the restrictive Courant-Friedrichs-Levy (CFL) condition due to the much smaller cut cells.

We resolve the problem of numerical integration by modifying the so-called *moment-fitting* approach [1]. The resulting hierarchical moment-fitting (HMF) [2] strategy with Ansatz order P leads to integration errors that decrease with an experimental order of convergence (EOC) of at least $O(h^{P+1})$.

The size of cut cells can be magnitudes smaller than regular cells. We apply cell agglomeration to the smallest cells, which does not only remedy the CFL restriction [3] but also stabilizes the numerical scheme. In addition, we use a local time-stepping (LTS) algorithm based on an explicit Adams-Bashforth multirate time-integration scheme [4] to efficiently integrate the remaining cut cells in time.

As a first example, we consider an isentropic vortex in a circular domain which is defined through a quadratic level set function (see Figure 1). The corresponding h-convergence results depicted in Figure 2 clearly indicate that our DG scheme maintains the optimal order of convergence for problems without interfaces. In contrast to similar approaches (e.g, [3, 5]), this can be achieved without the introduction of quadrature sub-cells or the like.

In the second part, we will show that this finding also holds for the more complex test cases and flow geometries where we have to deal with very small and/or degenerate cells. These measures render the approach applicable to a broad range of problems, including the extension of the DG method to higher order multi-phase flow calculations.

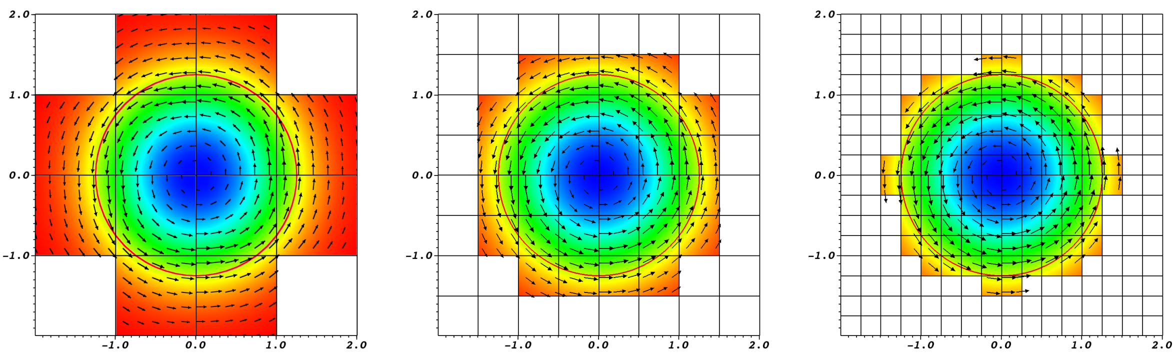
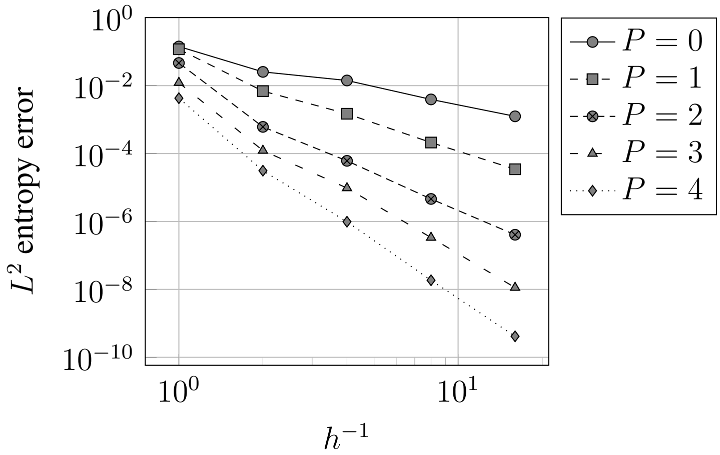


Figure 1: Velocity vectors and pressure distribution for an isentropic vortex in an ideal gas ($\gamma = 1.4$) where the physical domain is truncated at the zero iso-contour of a level set function (red circle). Here, the results for the three coarsest grids of the h-convergence study in Figure 2 are displayed.



Order	EOC
0	1.49
1	2.57
2	3.54
3	4.51
4	5.43

Figure 2: Results of the h-convergence study for a steady isentropic vortex in an ideal gas

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