

## Hierarchic *hp*-refinements for high-order finite elements

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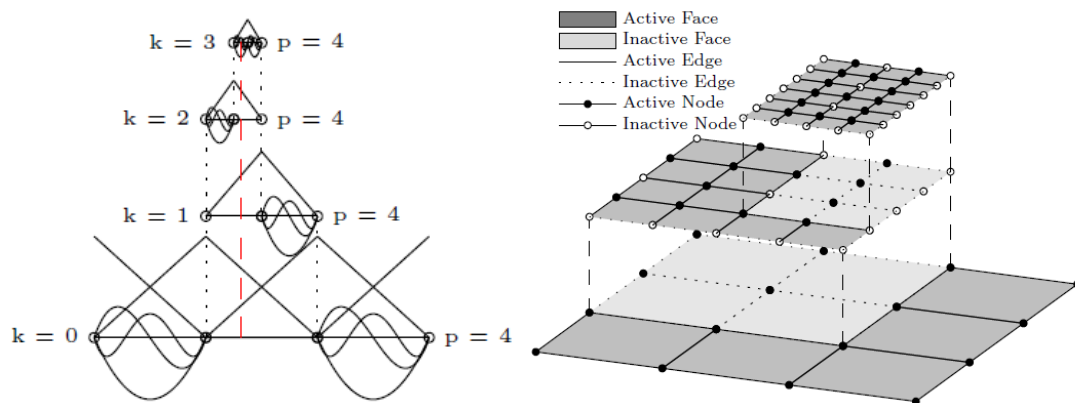
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*Key Words:* *hp*-FEM, Finite Cell Method, Embedded Domain Methods

### ABSTRACT

The implementation of *hp*-adaptivity is challenging as hanging nodes, edges and faces have to be constrained to ensure compatibility of the shape functions. For this reason, most *hp*-code frameworks restrict themselves to 1-irregular meshes in order to ease the implementational effort.

We will present the recently developed multi-level *hp*-refinement as a remedy to overcome these difficulties [1]. It provides full local *hp*-refinement capabilities at a comparably small implementational effort. Its main idea is the extension of the *hp-d* method [2] such that it allows for high-order overlay meshes yielding a hierarchical, multi-level *hp*-formulation of the Finite Element Method, see Figure 1.



**Figure 1:** Principle idea of multilevel-*hp* refinement in one and two dimensions [1]

This concept enables intuitive refinement and coarsening procedures, while linear independence and compatibility of the shape functions are guaranteed by construction. Multi-level *hp*-refinement is demonstrated to achieve exponential rates of convergence—both in terms of degrees of freedom and in run-time—also for problems with non-smooth solutions.

Furthermore, we present how multi-level  $hp$ -refinement can be used alongside the Finite Cell Method to simulate problems with complex topologies for which mesh generation would impose an additional, severe effort.

This new type of refinement leads to a slightly different Ansatz-space as compared to classical  $hp$ -refinements. We will point out similarities and differences in comprehensible examples.

## REFERENCES

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