

Spectral Properties and Conservation Laws in Mimetic Finite Difference Methods for PDEs

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ABSTRACT

The *Mimetic Finite Difference* (MFD) methods [1, 3] are numerical techniques to approximate the solutions of PDEs. The main idea of MFD methods is to mimic important properties of the continuous models, e.g., conservation laws, symmetry and positivity of the solutions, and the most important properties of the continuous differential operators, including duality and self-adjointness relations. Furthermore the MFD methods can be applied for general polygonal and polyhedral meshes instead of more standard triangular/quadrilateral (tetrahedral/hexahedral) grids. This feature has motivated a recent growth of interest in the engineering and numerical analysis literature.

In the talk we develop the analysis of the matrices involved in the MFD methods for linear diffusion problem with Dirichlet homogeneous boundary conditions. We analyse the sparsity and the spectral properties of the matrices and we derive the maximum and minimum eigenvalues, then the norm and the condition number of the matrices involved.

Furthermore we apply the MFD methods to parabolic problems. We consider as a model problem the classical time-dependent diffusion equation. We discuss the stiffness property of the semi-discrete system. We consider two time integrator schemes for solving the semi-discrete system: the θ -method, that, when $\theta > 0$, is an implicit method suitable for solving stiff ordinary differential equations (ODEs) and the *Runge-Kutta Chebyshev* (RKC) methods that are explicit methods but with large stability regions ([5]). We observe that RKC methods seem to be appropriate for parabolic problem where the spatial discretization is performed by MFD methods. The presence of stiff term is well treated by the stability properties of the RKC methods. Moreover, the explicit nature of the formulas limits the computational cost of the resulting linear systems.

Finally we treat the parabolic problems coupled with Neumann boundary conditions. By using the duality property and adjointness relations among the operators we can derive conservation laws for semi-discrete and fully discrete systems.

A large range of numerical tests in accordance with the theoretical derivations is presented.

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