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A fully-coupled Finite-Volume Method for Particulate Flow in Bio-Fluids

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ABSTRACT

A fully-coupled discretization method for the direct simulation of freely moving rigid particles in fluids is presented. The method shall be applied to simulate the motion of micro-spheres (d=40nm), so called synaptic vesicles. Their dynamics are mainly passively driven by small fluctuations of the surrounding, highly viscous, intra-cellular fluid. Therefore we pursue an accurate approximation of the bi-directional fluid-particle momentum coupling.

Our approach follows the key idea of the method of Glowinski et al. [1], who introduce *fictitious domains* by filling the rigid parts of the domain with a virtual fluid. Consequently, the Navier-Stokes equations govern the whole computational domain.
A widely used technique to enforce the rigid body constraint within the fictitious domain is the introduction of *Lagrange-Multipliers* for the rigid motion of the particles. A big drawback is a highly increased number of unknowns. Moreover numerical issues like stability of the resulting saddle point problem are not fully understood.

According to our discretization strategy, the interpretation of the Finite-Volume method as a Petrov-Galerkin Finite-Element discretization provides the theoretical foundation for the design of a consistent and stable method. In our *Finite-Volume-Element* method [2] the rigidity constraint is imposed by choosing appropriate approximation spaces satisfying the rigid body motion in the corresponding domain, as done for the X-FEM discretization by Wagner et al. [3]. As a consequence, our model equations purely consist of the original balance law for the linear momentum, being completed by balance law of the angular momentum in the case of rigid bodies.

A further challenge in the context of immersed boundaries is a preferably exact capturing of the interfaces, since they are not resolved by the underlying grid. Inspired by techniques used in the context of meshfree methods [4] we apply a *flat-top partition of unity*. The result is a consistent formulation of the discretization conforming to the immersed boundaries. Our technique can also be applied to more general model problems, in which boundary conditions need to be induced on a boundary not conforming to the computational grid.

We verified our method quantitatively by comparison with benchmark tests for a freely falling

particle in two and three dimensions. The results particularly confirm the improvement in accuracy, theoretically expected by the introduction of the *flat-top partition of unity*.

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