

The Nonconforming Virtual Element Method for the Convection-Diffusion-Reaction Equation

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ABSTRACT

In this talk, we present a new family of numerical schemes for solving the diffusion equation with convection and reaction terms in the primal form on unstructured polygonal and polyhedral meshes. These discretizations are built to satisfy local consistency and stability conditions. The consistency condition is an exactness property, i.e., these schemes are exact when the solution is a polynomial of an assigned degree. On its turn, the stability condition enforces the coercivity of the discrete bilinear form and, eventually, the well-posedness of the method.

According to [3, 4], we adopt a special choice of the degrees of freedom: we use solution moments on faces and inside cells with respect to the polynomial function up to a given degree. Higher order schemes are built using higher order moments, i.e., by increasing the degree of the polynomials.

These new kind of schemes are suitable to the numerical approximation of two- and three-dimensional elliptic problems at any order of accuracy on an arbitrary polygonal or polyhedral mesh. The developed schemes are verified numerically on diffusion problems with constant and spatially variable (possibly, discontinuous) tensorial coefficients. The casting of these scheme in the virtual element setting [1, 2] makes it possible to perform the convergence analysis and derive error estimates in the L^2 and the energy norm. It is worth mentioning that this kind of approximation shares many characteristics with the non-conforming finite element method, and, for this reason, we might refer to it as the "non-conforming virtual element method."

REFERENCES

- [1] B. Ayuso de Dios, K. Lipnikov, G. Manzini. The nonconforming virtual element method. *arXiv:1405.374 [math.NA]* (2014).
- [2] L. Beirao da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo. Basic principles of virtual element methods. *Math. Models Methods Appl. Sci.* 23:119–214 (2013).

- [3] L. Beirao da Veiga, K. Lipnikov, and G. Manzini. Arbitrary-order nodal mimetic discretizations of elliptic problems on polygonal meshes. *SIAM J. Numer. Anal.* 49(5):1737–1760 (2011).
- [4] K. Lipnikov and G. Manzini. A high-order mimetic method on unstructured polyhedral meshes for the diffusion equation. *J. Comp. Phys* 272:360-385 (2014).