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hp–Version Discontinuous Galerkin Methods on Polytopic Meshes

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ABSTRACT

The numerical approximation of partial differential equations (PDEs) posed on complicated domains which contain 'small' geometrical features, or so-called micro-structures, is of vital importance in engineering applications. In such situations, an extremely large number of elements may be required for a given mesh generator to produce even a 'coarse' mesh which adequately describes the underlying geometry. With this in mind, the solution of the resulting system of equations emanating, for example, from a finite element discretization of the underlying PDE of engineering interest on the resulting 'coarse' mesh, may be impractical due to the large number of degrees of freedom involved. Moreover, since this initial 'coarse' mesh already contains such a large number of elements, the use of efficient multi-level solvers, such as multigrid, or domain decomposition, using, for example, Schwarz-type preconditioners, may be difficult, as an adequate sequence of 'coarser' grids which represent the geometry are unavailable.

In recent years, a new class of finite elements, referred to as Composite Finite Elements (CFEs), have been developed for the numerical solution of partial differential equations, which are particularly suited to problems characterized by small details in the computational domain or micro-structures; see, for example, [6, 5], for details. This class of methods are closely related to the Shortley-Weller discretizations developed in the context of finite difference approximations, cf. [7]. The key idea of CFEs is to exploit general shaped element domains upon which elemental basis functions may only be locally piecewise smooth. In particular, an element domain within a CFE may consist of a collection of neighbouring elements present within a standard finite element method, with the basis function of the CFE being constructed as a linear combination of those defined on the standard finite element subdomains. In this way, CFEs offer an ideal mathematical and practical framework within which finite element solutions on (coarse) aggregated meshes may be defined.

In this talk, we consider the generalisation of CFE schemes to the case when hp–version discontinuous Galerkin composite finite element methods (DGCFEMs) are employed, cf. [1, 2]. In particular, we propose a new interior penalty (IP) scheme characterized by a careful choice

of the discontinuity-penalization parameter, which permits the use of polygonal/polyhedral elements such that

- mesh element faces may have arbitrarily small measure in two dimensions;
- both mesh element faces and edges may have arbitrarily small measure in three dimensions.

The approach is based on exploiting a new inverse inequalities relevant to elements with elemental interfaces whose measure is potentially much smaller than the measure of the corresponding element, cf. [3, 4]. On the basis of these inverse inequalities, together with appropriate approximation results on general polygons/polyhedra, we derive *a priori* error bounds for the proposed IP DGCFEM for general classes of second–order partial differential equations with nonnegative characteristic form. Furthermore, the application of this class of methods within general agglomeration–based refinement algorithms will be considered, based on employing dual–weighted–residual *a posteriori* error estimation techniques.

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