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XFEM/Phase Field methods for crack propagation in brittle materials

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ABSTRACT

We present a comparison of the two main approaches used to model and simulate crack propagation in brittle materials. On one hand, the XFEM method, introduced in [2] and [4], which allows for the discontinuity of the displacement field across the crack, on the other hand the variational approach, proposed in [5] and inspired by the work by Griffith [6], which introduces a smooth crack field that propagates along a path of least energy.

The XFEM is an efficient method to compute the displacement field for one or more fractures whose geometry is given. However, when it comes to deal with crack propagation, a rigorous theory to predict whether a crack will propagate or initiate, and the orientation of the new crack segments, is still missing. In the literature one may in fact find many different criteria, as reviewed in [10], but it is still unclear how to determine the velocity of the propagation [7]. All the proposed criteria rely on an estimation of the stress intensity factors, which can be done in 2D through the computation of the J-integral [11], as in [2] and [4]. Moreover, at the numerical level, the XFEM method requires algorithms to track the crack paths, which can be inefficient when dealing with a high number of fractures or with complex paths.

The alternative approach overcomes the difficulty of tracking the crack path and avoids relying on artificial criteria for propagation. It introduces an energy to be minimized with respect to the admissible displacements and to the fracture itself. The actual functional that is minimized in practice is a Γ-convergent approximation of that energy, see [1], that regularizes the displacement field across the fracture thanks to a phase field variable that interpolates between the unbroken and the broken states of the material. The irreversible character of the process can be accounted for in a thermodynamically compliant variant of the model, see [8, 9]. The crack path is a natural outcome of the analysis, see [3], but classical FEM can be applied with the heavy restriction that the mesh be fine enough to capture the high gradient of the phase field variable, which implies a high computational cost.

With the aim of efficiently simulating crack propagation in brittle materials, featuring phenomena such as initiation and branching in complex topologies, a promising strategy is to combine the advantages of the XFEM method and the variational approach. We compare in terms of accuracy and efficiency the solutions obtained with the two methods in some traditional test

configurations. The final aim is integrating the two procedures to allow both an accurate description of crack propagation and an efficient solution of the displacement problem.

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