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Multiscale coupling approach for solving high-dimensional stochastic problems featuring localized uncertainties and non-linearities

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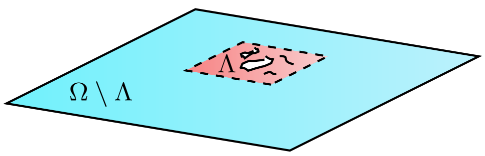
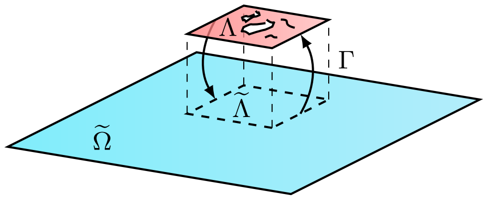
ABSTRACT

During the last two decades, functional approaches for uncertainty propagation have received a growing interest and are nowadays used to solve complex stochastic equations with multiscale features. Such stochastic multiscale models can exhibit variabilities in the operator, the boundary conditions or the geometry at different scales. Classical monoscale approaches are computationally demanding. Alternatively, multiscale approaches based on patches allow to capture the high complexity of solutions by introducing a scale separation. A multiscale coupling strategy dedicated to stochastic problems involving localized uncertainties has recently been proposed in [1]. It is based on an overlapping domain decomposition method leading to a global-local (two-scale) formulation of the stochastic problem. The associated global-local iterative algorithm involves successive solutions of simple global problems (with deterministic operator) defined over a deterministic domain and of complex stochastic local problems (with uncertain operator, right-hand side or geometry) defined over subdomains of interest (patches). Figure 1 provides a comprehensive sketch of the multiscale coupling strategy applied to stochastic multiscale problems featuring localized uncertainties. Convergence and robustness properties of the algorithm have been analyzed in [1] for linear elliptic stochastic problems.



(a) Random domain Ω and patch Λ containing localized uncertainties.



(b) Global model over fictitious domain $\widetilde{\Omega} = (\Omega \setminus \Lambda) \cup \widetilde{\Lambda}$ and stochastic local model over patch Λ coupled through deterministic interface Γ.

Figure 1: Non-intrusive coupling strategy applied to stochastic multiscale problems with localized uncertainties.

In the present work, the method is extended to non-linear elliptic stochastic problems with localized uncertainties and non-linearities. Convergence of the global-local iterative algorithm is proven for a class of non-linear elliptic stochastic partial differential equations. The flexibility and non-intrusivity of the multiscale coupling approach allow to consider not only different kinds of models but also different types of approximation spaces and solvers at global and local levels. In this work, stochastic local problems are solved using a least-square polynomial approximation which only requires the availability of standard deterministic codes and are well adapted to massively parallel computation without any software developments. Here, we rely on greedy algorithms proposed in [2] for the adaptive construction of sparse polynomial approximations. The accuracy of local approximate solutions is controlled by an adaptive selection of approximation spaces and of the number of samples for a stable approximation. The capability of this adaptive strategy to capture sparse high-dimensional polynomial approximations of local solutions is explored through numerical investigations. The performances of the multiscale domain decomposition method are illustrated through numerical experiments carried out on a diffusion stochastic problem with localized random material heterogeneities.

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