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# Volumetric locking in Local maximum entropy approximations

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## ABSTRACT

In numerical solution of partial differential equations, the presence of locking can lead to totally erroneous solution. In particular, for nearly incompressible or incompressible limit, it is a well-known phenomenon that Galerkin based formulation results in unstable solution due to volumetric locking. For example, in the analysis of stokes problem, the approximations chosen for velocity and pressure fields should satisfy the mathematical formulation with the additional divergence constraint. But not all choices of the approximation spaces fulfil this condition and cause instability in the numerical solution. This is called volumetric locking and stability of such formulation is usually characterized by Ladyzhenskaya Babuska Brezzi (LBB) condition. Analytically to prove this condition is cumbersome and this lead to the development of numerical test proposed by Chapelle-Bathe [1]. The finite discretization spaces that pass this numerical test are likely to satisfy inf-sup (LBB) condition.

In finite element method various techniques was proposed to alleviate locking. But for meshless methods it is a growing subject of interest, considering the evolution of meshless methods being used in automotive industries to simulate wide variety of problems like fluid structure interaction, free surface flows and applications that involves multiphysics like injection moulding process. Among meshless methods, a very recent approach is Local maximum entropy (Maxent) approximation technique [2] which is gaining popularity due to its advantages like smoothness and exact interpolation on the boundaries when compared to

that of its earlier counterparts. The smooth transition of basis functions from global, $C^{\infty}$ to local, $C^{0}$ is controlled by set of non negative parameter $\beta = \left\{\beta_a = \frac{\gamma_a}{(h_a)^2}\right\}_{a=1,\ldots,N}$, where $h_a$ is the nodal spacing (considered uniform in this case) and $\gamma_a$ is the dimensionless support width parameter that characterize the degree of locality of the basis function.

The behaviour of Maximum entropy meshless methods in incompressible limit will be discussed. The inf-sup test as proposed by Chapelle-Bathe is verified for Local maximum entropy approximations. As in most of other finite approximation schemes local Maxent approximations also fails inf-sup test, hence a numerical procedure based on partition of unity enrichment is proposed. This formulation proposed is aimed at solving this issue of volumetric locking in Stokes problem for nearly incompressible or incompressible limit.

## REFERENCES

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