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# Robust implementation of X-FEM with quadratic elements

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## ABSTRACT

The original design of the X-FEM formulation [1] helps to understand its huge popularity particularly with linear elements: X-FEM adds extra dofs at each node to model “a priori” known non-smooth solutions, matching with the support of the node. It has been demonstrated thoroughly [3] that this design recovers optimal rate of convergence. Nevertheless, conditioning and accuracy issues arise when the order of finite element interpolation increases (this is almost unseen with linear elements). In fact, some studies with 3D linear elements pointed out those limits of X-FEM design [2], but suggested that a good behavior could be restored through the use of numerical criteria. With quadratic elements, those limits can’t be address easily: numerical criteria used commonly with linear elements, failed systematically with quadratic elements [4]. Instead of working on a more complex criterion to “patch-up” the enrichment, we decided to put in question the design of the enrichment itself. During the talk, we will address the following concern: how to model strong discontinuities and cracks in 3D with quadratics elements through a robust design of X-FEM?

Basically, quadratic elements are very sensitive to redundant information in the approximation of the enrichment functions. A great care has to be taken in the design of each enrichment function whether modeling a strong discontinuity or a singularity.

The first step is to improve the design of enrichment function for strong discontinuities. X-FEM jump function has a blatant weakness: the sensitiveness towards the positioning of the discontinuity, particularly in 3D [2]. Studies have shown that a “special treatment” (usually named fit-to-vertex) is needed even with linear elements, when the interface comes close to the nodes [2][5]. With quadratic elements, the condition number deteriorates so badly that no “patching-up” criteria are reliable enough to ensure good conditioning and accuracy of the enrichment [4]. Based on [11], a new enrichment has been developed [4]. The new enrichment outmatches by far the accuracy and the robustness of previous X-FEM enrichments.

The second step is to improve integration, by taking into account the crack curvature. There are many algorithms in 2D [6] and advanced partitioning algorithms for 3D linear elements [7]. Nevertheless, they don’t extend easily to 3D quadratic elements given the curvature of the discontinuity’s surface and the use of quadratic elements. A middle nodes positioning algorithm has been developed to catch multi-directional curvature [4].

The last step is to improve the design of the singular enrichment. X-FEM singular enrichment introduces many dofs (12 dofs per node) in a fixed area around the crack, to reach optimal accuracy [9]. With quadratic elements, conditioning worsens very quickly [3]. Based on [8], we implement a

robust enrichment with less dofs (3 dofs per node), to model the singularity at the vicinity of the crack. To reach optimal accuracy, we combined it with a fixed area enrichment technique.

To sum up, this communication will address the weak points of X-FEM, in order to increase its robustness with respect to quadratic elements. Through numerous improvements, our "revised" X-FEM enrichment isn't sensitive anymore to conditioning issues when modeling cracks with quadratic elements in 3D (and 2D eventually).

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