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Stabilization and preconditioning of XFEM discretizations for 3D incompressible two-phase flows

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ABSTRACT

Two-phase systems play an important role in chemical engineering, for example mass transport between droplets and a surrounding liquid (liquid-liquid system) or heat transfer in falling films (liquid-gas system). The velocity and pressure field are smooth in the interior of each phase, but undergo certain singularities at the interface Γ between the phases. Surface tension induces a pressure *jump* across Γ, and a large viscosity ratio leads to a *kink* of the velocity field at Γ, especially for liquid-gas systems.

If interface capturing methods (like VOF or level set techniques) are applied, the finite element grid is usually *not aligned with the interface*. Then for standard FEM the approximation of functions with such singularities leads to poor $\mathcal{O}(\sqrt{h})$ convergence. The application of suitable extended finite element methods (XFEM) provides optimal approximation properties [2, 5], essentially reducing spurious currents at the interface. Figure 1 shows the pressure jump of a static bubble using a standard and an extended finite element space.

Besides of its optimal approximation properties, unfortunately, the linear systems arising from XFEM discretizations can be highly ill-conditioned because of a deterioration of the inf-sup constant and a lack of robustness w.r.t. the relative location of the interface inside cut elements. Recently, a couple of stabilization techniques have been proposed to attenuate these drawbacks [1, 4].

Figure 1: Pressure jump of a static bubble using piecewise linear FEM (left) and suitable XFEM (right).

In this talk we consider 3D incompressible flow simulations of such two-phase systems on adaptive multilevel tetrahedral grids. We present an XFEM space which is very suitable for the approximation of discontinuous pressure in incompressible two-phase flow problems and study the stabilization properties of a ghost penalty method applied in that context. The usual Cahouet-Chabard-type preconditioners for the Schur complement fail in the stabilized case, but can be modified to provide optimal preconditioning properties. We introduce such novel preconditioners both for the stationary and non-stationary stabilized two-phase Stokes equations and discuss their robustness w.r.t. numerical parameters (e.g., grid size, time step size, choice of the stabilization parameter) and material parameters (e.g., jump of the viscosity coefficient). At the end of the talk, we present application examples of 3D droplet and falling film simulations obtained by our 3D two-phase flow solver DROPS [3, 6].

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