Non-intrusive coupling: multiscale computation and finite element mesh adaptation

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ABSTRACT

In this paper, we present an iterative coupling algorithm for solving multiscale problems, in a non-intrusive way (*cf.* [4, 5]).
Let us consider a global linear finite element model on $\Omega = \Omega_G \cup \Omega_{\widetilde{G}}$, and a local (potentially non-linear) model on Ω_L (*cf.* Fig. 1). The purpose of such algorithm is to replace the linear model on the area of interest $\Omega_{\widetilde{G}}$ by the local model Ω_L, without altering the numerical operator associated with the initial global model on Ω (for example the stiffness matrix K).

Starting from an initial solution $U^{(0)}$, the algorithm consists in a fixed point which tends to provide reaction equilibrium between the global and the local models at the interface (*i.e.* $\mathcal{F}_{\Omega_G/\Omega_{\widetilde{G}}} + \mathcal{F}_{\Omega_L/\Omega_G} = 0$) while verifying mechanical equilibrium over Ω and Ω_L respectively.

Non-matching meshes at the interface are handled by enforcing displacement continuity in a weak sense (using a mortar method), allowing for a flexible multiscale and/or multimodel coupling.
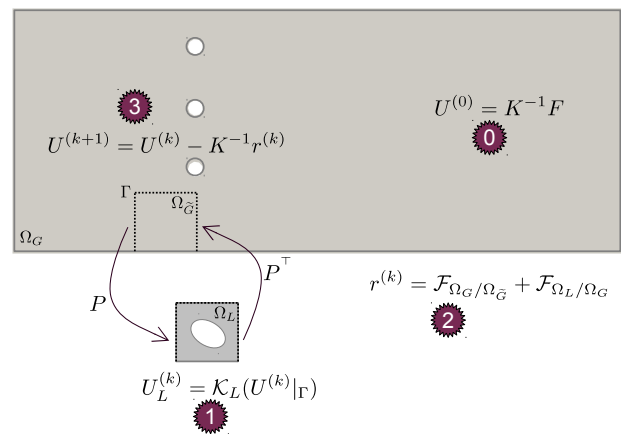

Figure 1: Non-intrusive coupling algorithm

Finally, such algorithm is applied to automatic mesh adaptation, in a non-intrusive way, *i.e.* allowing for refinement with patches acting as a replacement of the initial mesh on localised areas.

An a posteriori error estimation procedure is used to ensure the proper placement of the mesh-refined patch (*cf.* [3]). We rely on explicit residual error estimates (*cf.* [1]) which are very cheap to compute, while providing relatively good elementary error distribution.

The main property of such algorithm is its non-intrusiveness, *i.e.* the fact that the initial global mesh remains unaffected. Moreover, the algorithm depicted at Fig. 1 can be optimised using Quasi-Newton acceleration. Then, only a few iterations are required in order to reach equilibrium (*cf.* Fig. 2).

(a) Convergence speed
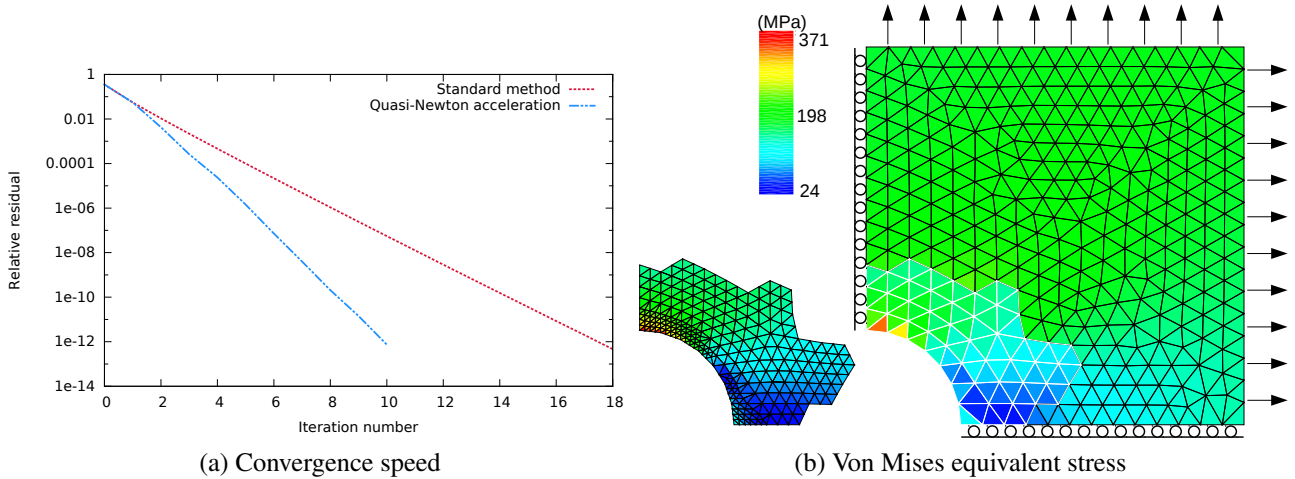
(b) Von Mises equivalent stress

Figure 2: Non-intrusive coupling: application to local mesh refinement

Then, the error on the non-intrusive composite (multiscale) solution itself is also to be assessed in order to guarantee the reliability of the method.

The classical explicit residual formulation has to be adapted due to the non-conforming (nested) interface at the patch edge, using the properties of Scott-Zhang interpolation operator (*cf.* [2]) to assess the interpolation error.

Such method allows to easily sew different mesh types (triangle/quadrangle) and orders (*h-p* refinement).

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