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High order discretizations for two-phase flow: spatial discretization, curvature evaluation and solver strategies

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ABSTRACT

We are going to present a high-order discretization for two-phase flow problems, based on an extended discontinuous Galerkin (DG) discretization. This method combines a classical DG method with an interface treatment similar to the extended Finite Element methods (XFEM). The support of the basis functions is conformal with the fluid interface between the two phases. Therefore the 'h^p-convergence' property of the DG method can be achieved even for low-regularity, discontinuous solutions which appear in two-phase flows, as shown in figure 1; convergence plots are shown in figure 2.

The intersection of the fluid interface with a fixed grid may produce very small cut cells, which have a disastrous effect on the condition number of the (linearized) saddle point problem that arises from the discretization. These issues can be overcome using a cell agglomeration technique.

Surface tension causes a jump in the pressure field (see figure 1) at the interface which is proportional to the curvature of the interface. In our framework, this interface is given by a level-set. The curvature can then be computed from first- and second-order derivatives of this level-set function φ. During the time-evolution of φ it is usually not possible to conserve continuity of φ and its derivatives, even if the initial data is $\mathcal{C}^\infty$, which is a source of instability.

We present a stabilized curvature evaluation process, see [1]. The algorithm is based on a patch-recovery process, to regain approximate continuity of higher derivatives of φ. Combined with a stabilized level set movement,the stability issues can be overcome. We carried out extensive benchmarking to provide a balance between accuracy and computational cost.

The nonlinear saddle point problem resulting from the discretization can be solved using different techniques, including e.g. rather classical approaches like the SIMPLE algorithm (see [2]), or Krylov-acceleration techniques originally proposed by [3].

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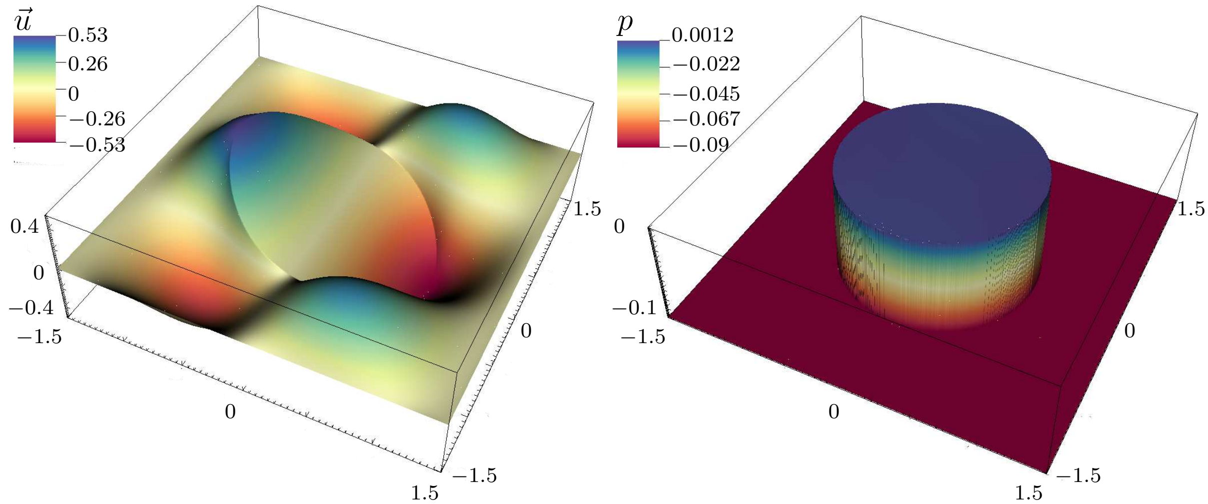

Figure 1: Velocity in x-direction (left side), pressure (right side) and velocity vectors/interface position/grid (bottom) for the steady ellipsoidal droplet.

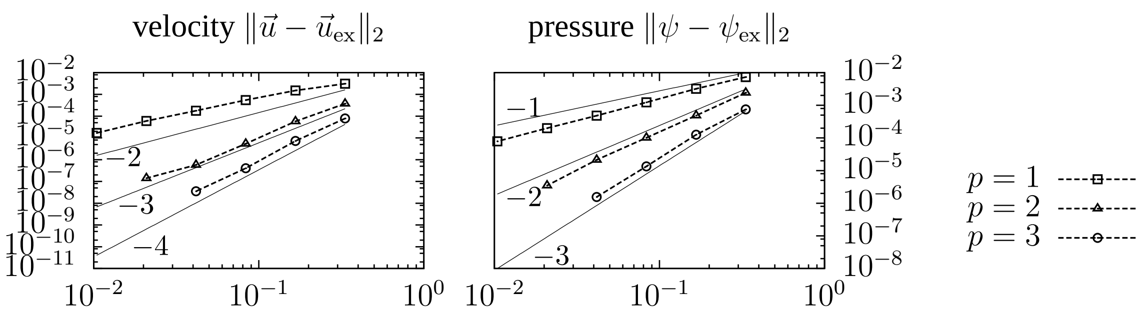

Figure 2: Convergence study on velocity (left side, polynomial degree k) and pressure error (right side, polynomial degree $k' = k-1$). Horizontal axis: logarithm of grid resolution, i.e. $\log_{10}(h)$, where h denotes the grid resolution. Vertical axis: logarithm of error in the L^2-norm, i.e. $\log_{10}(\|\vec{u} - \vec{u}_{ex}\|_2)$ resp. $\log_{10}(\|p - p_{ex}\|_2)$.

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