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Basic principles of hp-Virtual Element Methods

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ABSTRACT

Virtual Elements are a recently introduced (see e.g. [3] and [4]) Galerkin method for the approximation of the solution of PDEs. Tipically, the local discrete spaces contain polynomials and, differently from the Finite Element framework, also other functions which are not known explicitly. The main features of VEM are the employment of very general types of meshes, i.e. polygonal/polyedral and even non conforming meshes and the fact that they can also be used in order to build global discrete spaces of arbitrary regularity.

hp-Finite Elements are the classical Finite Elements in which convergence is achieved by means of both mesh refinement and the increasing of the polynomial degree. It is well known (see e.g. [6]) that hp-FEM are superior to classical h-FEM in some particular cases; for instance, they can achieve exponential convergence when the solution is piecewise analytic; besides, they are more robust in presence of locking phaenomena.

In the present talk, which is based on the pioneering work [5], we introduce an hp-Virtual Element method for the two dimensional Poisson problem in primal formulation. Both theoretical and numerical results are discussed; in particular, the new hp-estimates are compared to those for the hp-FEM (see e.g. [1] and [2]). Remarks on the conditioning of the stiffness matrix are also presented.

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