

Extensions of the Unfitted Discontinuous Galerkin method for coupled bulk-surface PDEs

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ABSTRACT

Modern imaging techniques yield high quality information of complex shaped microscopic structures. The unfitted discontinuous Galerkin method (UDG) offers an approach to solve PDEs on implicitly described sub-domains, e.g. obtained using micro-CT imaging, without the need to construct a geometry-resolving mesh. The domain description uses a level-set based formulation; still sub-domain boundaries are incorporated explicitly.

We discuss extensions of the UDG method to surface PDEs and their coupling to bulk equations. A narrow-band approach allows to improve efficiency and local mass conservation. To obtain a UDG constructs basis functions from a simple background mesh and restrict the support according to the actual sub-domain boundaries, i.e. the implicitly prescribed domain. Using the explicit reconstruction of the implicit sub-domain boundary it is possible to couple level-set based surface problems on the interface with sub-domain problems.

Possible applications cover a wide range of applied problems, especially from medicine and biology. Recent efforts were made to incorporate the UDG method into a general multi-physics extension to the DUNE PDELab library.