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Phase Field Approach to Fluid Filled Fractures using Unfitted Discontinuous Galerkin Methods

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ABSTRACT

One advantage of Discontinuous Galerkin (DG) methods is, that they easily adapt to complex domains. We use this flexibility to allow for actual discontinuities in our numerical solution along an evolving jump set $\mathcal{C}$. In combination with cut-cell methods it is feasible to resolve $\mathcal{C}$, even if it is not aligned with mesh faces.

In standard continuous Finite Element (FE) methods discontinuities in the solution can only occur along (inner) boundaries. Thus an evolving set $\mathcal{C}$ would require remeshing to resolve the discontinuities. To overcome this difficulty the discontinuity set of our solution is regularized by a phase field φ converging to $\mathcal{C}$ in the sense of Γ - limits. Including φ explicitly in the stabilization term of the DG formulation results in numerical solutions that are discontinuous along $\mathcal{C}$.

Restoring the jump set from the phase field is done by an extended Marching Cube algorithm that yields a piecewise linear reconstruction of $\mathcal{C}$, similar to the approach implemented in Unfitted DG methods. UDG methods were designed to handle difficult geometries without remeshing. They were already successfully applied to flow problems in porous media [1].

To test our method it is applied to the pressure driven propagation of fluid filled fractures in the subsurface. We follow the phase field approach proposed by Bourdin et al. [2] and Mikelic et al. [3] to regularize the crack surface. Based on their approach we use our DG formulation to take into account discontinuities of the displacement along the propagating fracture. As the phase field models a fracture here, it additionally has to satisfy an irreversibility condition. The TNNMG method [4] is used to ensure this property.

The methods are implemented and tested using the DUNE framework [5]. We compare our results to standard FE methods for some examples of pressure driven fracture propagation from the literature.

REFERENCES

[1] Bastian P., Engwer C., Fahlke J., Ippisch O. An Unfitted Discontinuous Galerkin Method for Pore-scale Simulations of Solute Transport. *Math. Comput. Simul.* 81(10). 2051–2061 (2011).

[2] Bourdin B., Francfort G. A., Marigo J.-J. Numerical experiments in revisited brittle fracture. *J. Mech. Phys. Solids*. 48(4). 797–826 (2000).

[3] Mikelic A., Wheeler M. F., Wick T. A phase field approach to the fluid filled fracture surrounded by a poroelastic medium. *ICES Report 13-15* (2013).

[4] Gräser C., Sack U., Sander O. Truncated nonsmooth Newton multigrid methods for convex minimization problems. *Domain decomposition methods in science and engineering XVIII*. Lect. Notes Comput. Sci. Eng. 70. 129–136 (2009).

[5] Bastian P., Blatt M., Dedner A., Engwer C., Klöfkorn R., Kornhuber R., Ohlberger M., Sander O. A generic grid interface for parallel and adaptive scientific computing. Part II: Implementation and tests in DUNE. *Computing* 82(2–3). 121–138 (2008).