On the application of a non-intrusive coupling strategy for the local enrichment of NURBS patches: geometrical details

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ABSTRACT

With the introduction of the IsoGeometric Analysis (IGA) [1], the computation of structures has become possible using the exact geometry of the Computer-Aided Design (CAD) model regardless of the mesh density. For that, Lagrange polynomials are replaced by Non-Uniform-Rational-B-Splines (NURBS) functions to perform the analysis. NURBS functions have a higher order of continuity, namely $C^{(p-1)}$ through the elements of the mesh for a polynomial degree p, which on a per-degree-of-freedom basis exhibits increased accuracy in comparison to standard Finite Element Methods (FEM). However, difficulties are still encountered to model local behaviors such as geometrical details in a NURBS patch. To answer the issue, new splines like T-Splines [2] may be used. Nevertheless, this technique can still appear complex to implement and additional efforts may be necessary to answer the issue of describing a local behavior different from the global one (inclusions). Furthermore, this approach may suffer from some intrusiveness: if one is interested in optimizing the shape and/or the number of the details, one would need to re-build and to re-compute the whole problem at each step.

In this context, we propose in this work to make use of a non-intrusive coupling strategy introduced in FEM [3] for the modeling of geometrical details (holes, inclusions, local fracture,...) in a NURBS patch. Let us take, as a first example, the situation of holes as geometrical details of a rectangular structure to explain the method (see Fig. 1(a) for illustration). The domain of the true structure, *i.e.* the plate including the holes, is denoted Ω_{11} . For the holes, we refer to domains Ω_{12} . Finally, domain $\Omega_1 = \Omega_{11} \cup \Omega_{12}$ is introduced to characterize the plate without the holes. In CAD programs, such a geometry is classified as a trimmed surface. Its description is given by: a B-Spline surface parametrization for Ω_1 and several NURBS curve parametrizations for the trimming curves that form the boundaries of each hole. As a consequence, a new NURBS surface parametrization that directly defines Ω_{11} needs to be built to perform the analysis. Such a task can be very complicated in practice. To overcome this difficulty, the idea of the proposed approach is to take the NURBS patch for domain Ω_1 as the global model, and a local "empty" submodel for the geometrical details meant to replace the global model in domains Ω_{12} . A degenerated version of the iterative non-intrusive algorithm of [3] is implemented for this substitution. During the iterations, the stiffness operator to be inverted is the global operator (over Ω_1): it remains unchanged (and well-conditioned), which can facilitate the optimization of the details. Non-conforming geometries need to be addressed (the boundaries of the holes are not aligned with the elements in Ω_1). For that, an exact NURBS domain is simply constructed from the NURBS trimming curves by adding multiple interpolatory control points at the center of the detail (see Fig. 1(a)). We emphasize that this NURBS mesh is only here to produce accurate integration rules to evaluate the reaction forces, the corresponding stiffness operator is never assembled and inverted. For the specific case of holes, our method have similarities with the NUBRS version of the Finite Cell Method (FCM) [4] but, with the advantages of non-intrusiveness and well-conditioned operators.

The results of the method when applied to a perforated tensile specimen are shown in Fig 1. With the meshes of Fig. 1(a), good displacements (Fig. 1(b)) and stresses (Fig. 1(c)) were obtained within few iterations, especially when accelerations techniques are used (Fig. 1(d)). Following the same idea, the method has also been developed for the modeling of local "material" models (inclusions, local fracture) and has appeared to be efficient as well.



Figure 1: Non-intrusive analysis a perforated strip under tension: rectangular B-spline mesh of quadratic 30×60 elements for the plate + circular NURBS meshes of quadratic 20×5 elements for the holes.

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