Multigrid algorithms for hp-version Discontinuous Galerkin methods on polygonal and polyhedral meshes

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ABSTRACT

We present multigrid algorithms for the efficient solution of the linear system of equations arising from hp-version discontinuous Galerkin discretizations of second-order elliptic problems on polygonal/polyhedral meshes. We prove that, under suitable assumptions on the agglomerated coarse grid, the two-level version of the method converges uniformly with respect to the granularity of the grid and the polynomial approximation degree p, provided that the number of smoothing steps, which depends on p, is chosen sufficiently large. An analogous result is then obtained for the multigrid algorithms, under an additional assumption on the maximum number of levels. Numerical experiments confirm the effectiveness of the proposed schemes. Moreover, we also demonstrate numerically that the proposed solvers are convergent in practice, even when some of the theoretical assumptions are not fully satisfied.