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A Virtual Element Method for some Structural Mechanics problems

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ABSTRACT

We present a Virtual Element Method (VEM) for non-linear elastic and inelastic problems, proposed and studied in [1]. We mainly focus on the small deformation regime and, more precisely, we will consider the following cases:

1. non-linear elastic constitutive laws in a small deformation regime which, however, pertain to stable materials;
2. inelastic constitutive laws in a small deformation regime as they arise, for instance, in classical plasticity problems.

However, we do not consider situations with internal constraints, such as incompressibility, which require additional peculiar numerical treatment.

Virtual elements, which by construction allow for general polygonal and polyhedral meshes (see [2], for instance), were introduced for the linear elasticity problem in [3, 4]. Our scheme is one of the very first developments of the VEM technology for nonlinear problems, and it is designed in such a way that a general non linear constitutive law can be automatically included. Indeed, on every element of the mesh the constitutive law needs only to be applied once (similarly to what happens in one-point Gauss quadrature scheme) and the constitutive law algorithm can be independently embedded as a self-standing black-box, as in common engineering FEM schemes. Therefore, the method is computationally efficient, in the sense that the constitutive law needs to be applied only once per element at every iteration step. The risk of ensuing hourglass modes is avoided by using an evolution of the standard VEM stabilization procedure used in linear problems.

Some theoretical results, as well as a number of numerical tests, will be presented.

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