

Application of the Meshless Natural Neighbour Galerkin Method

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ABSTRACT

In this work we present applications of the meshless natural neighbour Galerkin method [1] to strain gradient elasticity, phase field modelling, and the analysis of laminated composite plates. The strain gradient theory, for gradient continuum analysis has kinematic relations which include terms from second gradients of displacements [2]. This results in balance equations which have fourth order spatial derivatives analogous to Kirchhoff plate theory, together with higher order boundary conditions [5]. The weak formulations of fourth order operators are well defined and integrable, only if the basis functions are piecewise smooth and globally C^1 continuous. The C^1 continuous shape functions are shown in Figure 1. The Cahn- Hilliard equation is a fourth-order nonlinear partial differential equation that allows to model phase separation in binary mixtures.

The analysis of the laminated composite plates using Reissner-Mindlin-type plate theory only necessitates C^0 continuity; whereas the Bernoulli/Kirchhoff-type beam/plate/shell theories with zero shear strains require C^1 continuity. Such formulations can be numerically solved by using the natural element method. In this work we present the application of C^1 natural element method for the analysis of kirchhoff plates. We also present a geometrical nonlinear analysis of laminated composite plates using C^0 natural element method. A Generalized higher order shear deformation plate theory is adopted for composite plate analysis [3]. The geometric nonlinearity is based on the von Karman assumptions. The nonlinear static, free vibration and buckling analysis is carried out with the incremental step loading and by the Newton-Raphson iterative method. The formulation developed here is validated with available analytical and finite element results. The effect of plate aspect ratio on the deflection has also been studied.

There are a very limited number of finite elements in two dimensions, possessing C^1 continuity applicable to complex geometries and higher order boundary conditions. To address this, we propose a methodology for the analysis of gradient or generalized continua using the natural element method. C^1 natural neighbour interpolants are achieved by a simple transformation of the Farins interpolant, which are basically obtained by embedding Sibsons natural neighbour coordinates in Bernstein-Bezier surface representation of a cubic simplex [4]. Numerical examples will be presented to demonstrate the efficiency of the method.

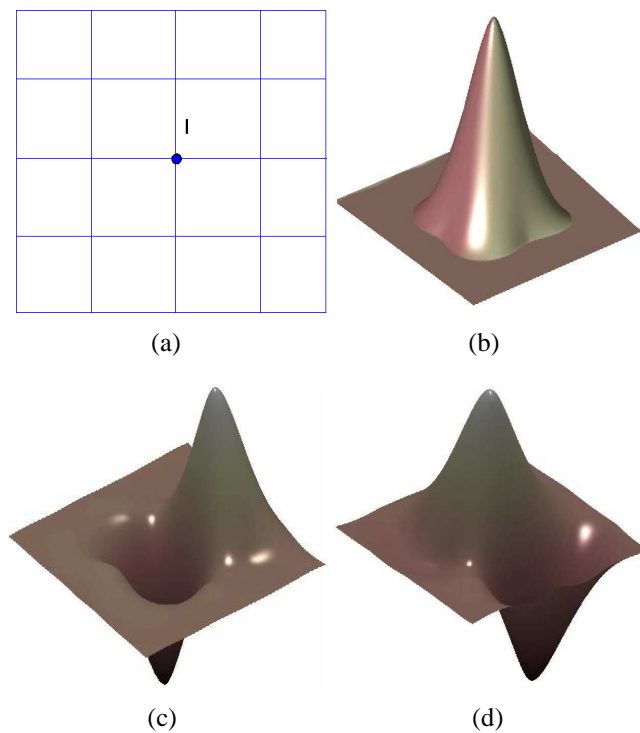


Figure 1: Illustration of natural element shape functions. (a) Grid, (b) C^1 -continuous shape function N_{3I-2} at node I , (c) C^1 shape functions N_{3I-1} , and (d) N_{3I} .

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