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An Adaptive Iso-Geometric Analysis for solving plane problems

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ABSTRACT

In the present work, an adaptive Iso-Geometric analysis [1] is presented for solving plane problems. The rate of convergence of the solution are studied by considering $h-$ and $rh-$ refinements. The choice of control mesh is arbitrary resulting in highest estimated error. One of the classical way to reduce the error is by studying the adaptive refinement strategy. The effect of such method relies on how accurately we measure the error in numerical solution. The classical approach is to estimate the point-wise interpolation error in Sobolev space: $\boldsymbol{e} = \boldsymbol{u} - \boldsymbol{u}^h$. There exist even other methods for error estimation, see [2, 3].

Point-wise interpolation error estimation is not direct in Iso-Geometric analysis as the solutions are not interpolatory functions. In the proposed method, we considered hierarchical B-spline function as sufficiently smooth function to be in Sobolev space. In this regard, the interpolation error estimated are acceptable as the continuity properties are retained.

Let $\tilde{u}^h \in \mathfrak{S}^h = \{\tilde{u}^h \in H^p(\Omega) \mid \tilde{u}^h = u_g \text{ in } \Gamma_u\}$ and $\tilde{u} \in \mathfrak{S}^h = \{\tilde{u} \in H^{n>p}(\Omega) \mid \tilde{u} = u_g \text{ in } \Gamma_u\}$ be the B-spline basis of function of different order

$$||e||_m = ||\tilde{u}^h - \tilde{u}||_m \tag{1}$$

The physical mesh obtained in IGA are only end interpolatory to control mesh. In such cases we define control variables where degrees of freedom, boundary conditions and geometry of physical domain are prescribed, unlike nodal variables in FEM. Such description pose issues in imposing strong Dirichlet boundary conditions, see [4, 5]. In the proposed method, the control mesh is obtained by doing recursive sub-division of reference mesh. There by making the physical mesh to exactly interpolate the control mesh. This allows the exact imposition of essential boundary conditions in the classical Iso-Geometric analysis, see Fig. 1. Spring analogy method is used for performing $r-$ refinement. Further, $h-$ and $rh-$ refinement studies are explored.

The numerical examples considered is block under pressure to demonstrate the proposed adaptive strategies. The convergence study made for both $h-$ and $rh-$ refinement are shown in Fig. 2. The studies of adaptive refinement concluded the following points: (a) The point-wise error estimation in Sobolev space indicates convergence of solution with increase in degrees of freedom. (b) The proposed method of $h-$ refinement illustrates a higher rate of convergence. (c)

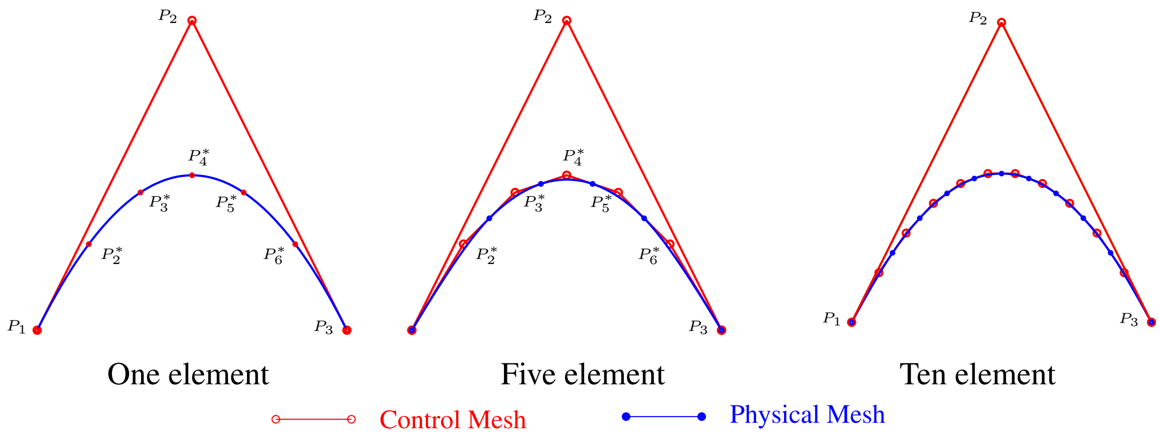

Figure 1: $h-$ refinement in IGA

Spring load method seeks for the geometry of control mesh to follow the geometry of the solution field in order to reduce the error. (d) The $rh-$ refinement shows better rate of convergence than only $h-$ refinement.

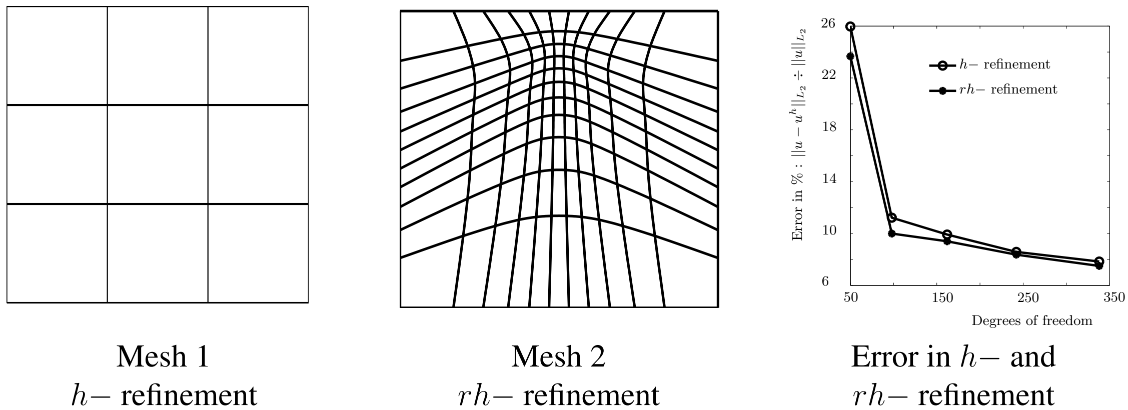

Figure 2: $h-$ and $rh-$ refinement meshes and error plot for block under pressure

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