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Convection-adapted BEM-based Finite Element Method on Tetrahedral and Polyhedral Meshes

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ABSTRACT

A new discretization method for homogeneous convection-diffusion-reaction boundary value problems in 3D is presented that is a non-standard finite element method with PDE-harmonic shape functions on polyhedral elements, see [1]. The element stiffness matrices are constructed by means of local boundary element techniques. The method, which is referred to as a BEM-based FEM, can therefore be considered a local Trefftz method with element-wise (locally) PDE-harmonic shape functions.

The current research combines the results of [2] with the hierarchical construction of shape functions presented in [3]. The Dirichlet boundary data for these shape functions is chosen according to a convection-adapted procedure which solves projections of the PDE onto the edges and faces of tetrahedral and polyhedral elements, respectively. This improves the stability of the discretization method for convection-dominated problems both when compared to a standard FEM and to previous BEM-based FEM approaches, as we demonstrated in several numerical experiments.

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