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A new formulation for imposing Dirichlet boundary conditions on non-matching meshes

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ABSTRACT

Generating matching meshes for problems with complex boundaries is often an intricate process, and the use of non-matching meshes appears as an appealing solution. Yet, enforcing boundary conditions on non-matching meshes is not a straightforward process, especially when prescribing those of Dirichlet type. By combining the IGFEM (a type of GFEM described in [1]) with the Lagrange multiplier method, we introduce a new approach for the treatment of essential boundary conditions on non-matching meshes. The new formulation yields a symmetric stiffness matrix and is straightforward to implement. As a result, the methodology makes possible the analysis of problems with the use of simple structured meshes, irrespective of the problem domain boundary. Through the solution of linear elastic problems, we show that the optimal rate of convergence is preserved for piecewise linear finite elements. Yet, the formulation is general and thus it can be extended to other elliptic boundary value problems.

As an example, consider the classical Eshelby problem shown in Figure 1a, where a circular matrix with properties E_1, ν_1 contains a circular inclusion with properties E_2, ν_2. With boundary conditions $u_r(r_u, \phi) = r_u$, and $u_\theta(r_u, \phi) = 0$, the exact solution for the displacement field is given by

$$u_r(r,\phi) = \begin{cases} \left[\left(1 - \frac{r_u^2}{r_i^2}\right)\alpha + \frac{r_u^2}{r_i^2}\right] r & \text{for} \quad 0 \leq r \leq r_i, \\ \left(r - \frac{r_u^2}{r}\right)\alpha + \frac{r_u^2}{r}, & \text{for} \quad r_i < r \leq r_u, \end{cases}$$
$$u_\theta(r,\phi) = 0,$$

where α is a function of the geometry and the Lamé constants for the matrix (λ_1, μ_1) and for the inclusion (λ_2, μ_2):

$$\alpha = \frac{(\lambda_1 + \mu_1 + \mu_2) r_u}{(\lambda_2 + \mu_2) r_i^2 + (\lambda_1 + \mu_1)(r_u^2 - r_i^2) + \mu_2 r_u^2}.$$

Figures 1b and 1b show the type of meshes used to perform the analysis. The difference between the two is that the latter does not match to either the interface nor to the external boundary.

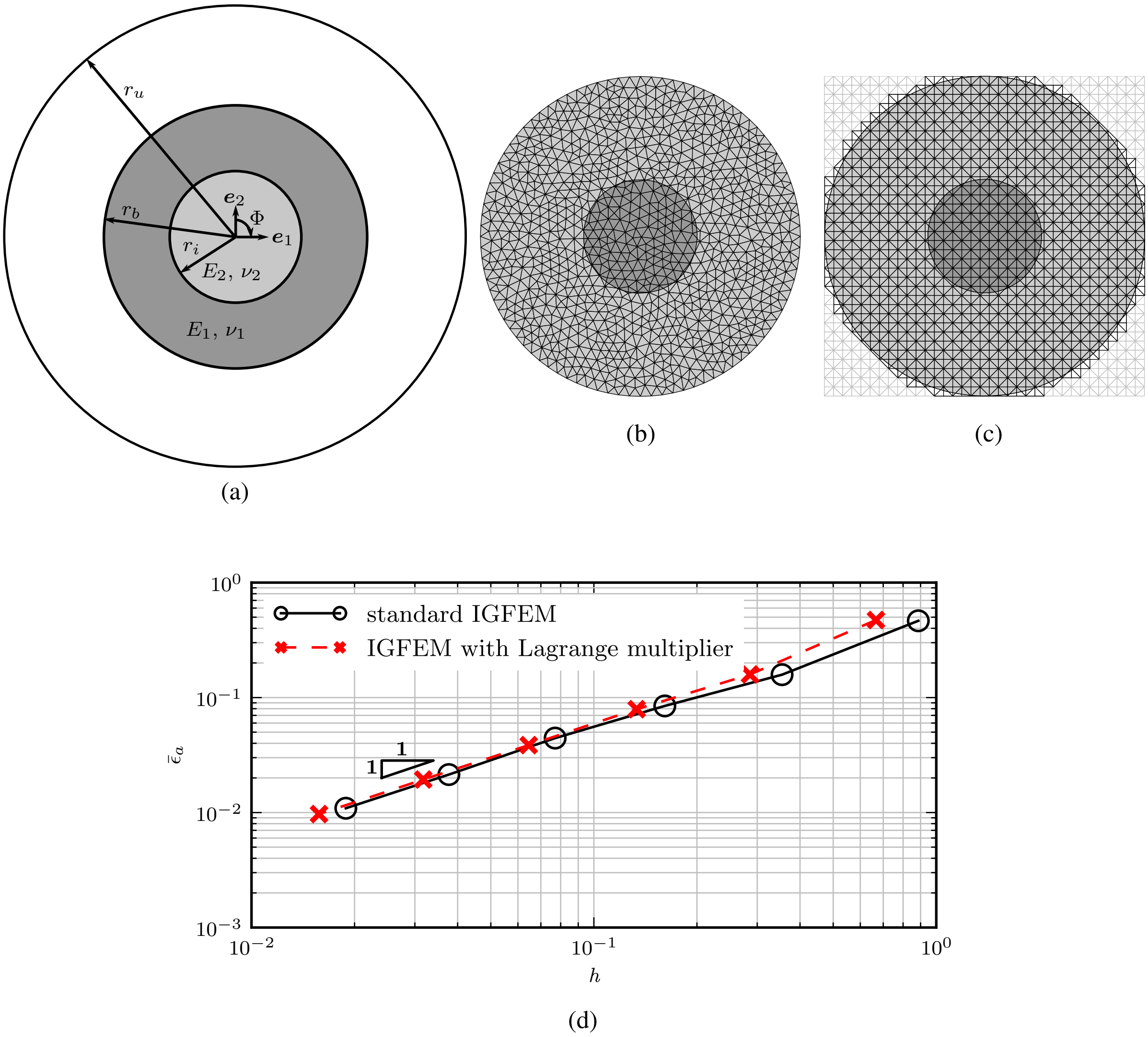

Figure 1: (a) Eshelby inclusion problem schematic; (b) Unstructured mesh that conforms to the Dirichlet boundary for the standard IGFEM method; (c) Structured non-conforming mesh for the IGFEM solution coupled with Lagrange multipliers; and (d) Convergence results showing in ordinates the energy norm as a function of the mesh size h.

Convergence results in the energy norm are given in Figure 1d, where it is demonstrated that the new methodology is able to recover optimal rates of convergence.

REFERENCES

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